## GDC REVISION for MATH SL for the CASIO model <br> by Christos Nikolaidis

From [MENU] we use
A. RUN-MAT
B. EQUA
C. STAT
D. GRAPH
E. RECUR (for sequences)

- Remember
- to use RAD or DEG appropriately in SETUP (shift-menu)
- in STAT - CALC to SET the first two lines

LIST1-LIST2 if you have frequencies

- in STAT - DIST we select

Data : Variable instead of List
For NORMAL we use $\quad$ Npd - Ncd - InvN $\quad$ (never Npd)
For BINOM we use Bpd-Bcd-InvB (never InvB)

- in SETUP (shift-menu) select Derivative On
- Common mistakes

| For | $x-3$ | we must use the difference - and not the symbol $(-)$ |  |
| :--- | :--- | :--- | :--- |
| For | $\sin ^{2} x$ | we must write | $(\sin x)^{2}$ |
| For | $\sin 3(x-2)$ | we must write | $\sin (3(x-2))$ |

In calculus, when we deal with sin, cos etc we must use rad

## A. EXAMPLES FOR RUN-MAT

1. Use $[M A T H]$ to find the following
(a) $\log _{2} 5$
(b) $f^{\prime}(2)$ and $f^{\prime \prime}(2)$ (by using $\frac{d}{d x}$ and $\frac{d^{2}}{d x^{2}}$ ) for

$$
f(x)=\frac{2 x+5}{3 x-7}
$$

(c) the definite integral

$$
\int_{3}^{5} \frac{2 x+5}{3 x-7} d x
$$

(d) the sum

$$
\sum_{k=7}^{20} \frac{2 k+5}{3 k-7}
$$

(e) the sum of the multiples of 7 between 1 and 200

## Answers

(a) 2.32
(b) -29 and -174
(c) 5.80
(d) 14.07
(e) Since 200/7=28.6 and the multiples of 7 have the form $7 x$ :

$$
\sum_{x=1}^{28} 7 x=2842
$$

2. Use [OPTION] and [PROB] to find
(a) $12!\quad$ (b) $\binom{12}{5}$

## Answers

(a) 479001600
(b) 792
3. Use [OPTION] and [NUM]-[Abs] to find
(a) the definite integral

$$
\int_{3}^{5}\left|t^{2}-16\right| d t
$$

(b) the sum

$$
\sum_{k=1}^{30}|100-6 k|
$$

## Answers

(a) 8
(b) 1358
4. Use $[O P T I O N]-[C A L C]-[S o l v e N]$ to solve the equations
(a) $e^{x}=2 x+3$
(b) $\sin 3 x=0.5 \quad 0 \leq x \leq \pi$
(c) $\sin 3 x=0.5 \quad 0^{\circ} \leq x \leq 180^{\circ}$

## Answers

(a) SolveN $\left(e^{x}=2 x+3\right)$ gives: $-1,37,1,92$

Notice: We can add a restriction for the domain. Use rad or degrees appropriately:
(b) SolveN $(\sin 3 x=0.5, \mathrm{x}, 0, \pi)$ gives: $\pi / 18,5 \pi / 18,13 \pi / 18,17 \pi / 18$
(c) $\operatorname{SolveN}(\sin 3 x=0.5, \mathrm{x}, 0,180)$ gives: $10,50,130,170$

## B. EXAMPLES FOR EQUA

5. Use [F2:Polynomial] to solve $x^{3}-5 x^{2}+3 x+6=0$

## Answer

$3.79,2,-0.791$
6. Use [F1:Simultaneous] to solve the simultaneous equations

$$
\begin{aligned}
& 3 x+5 y=18 \\
& 7 x-4 y=-5
\end{aligned}
$$

## Answer

$x=1, y=3$
7. Use [F1:Simultaneous] to solve the simultaneous equations

$$
\begin{gathered}
3 a+6 b-c=19 \\
a-2 b+4 c=3 \\
7 a-13 c=-19
\end{gathered}
$$

## Answer

$a=1, b=3, c=2$

## C. EXAMPLES FOR STAT

8. STATISTICS: Use [STAT] - [CALC] - [SET] - [VAR1]
A. Consider the data

$$
2,5,7,5,3,2,5,1,7,9
$$

to find the following:

| Mean = 4.6 | Range $=$ max-min $=\mathbf{9 - 1}=\mathbf{8}$ |
| :--- | :--- |
| Median $=\mathbf{5}$ | Interquartile range $=$ Q3- Q1 $=\mathbf{7 - 2}=\mathbf{5}$ |
| Mode $=\mathbf{5}$ | Standard deviation $=\sigma=\mathbf{2 . 4 5 8}$ |
| Lower quartile $=$ Q1 = 2 | Variance $=\sigma^{2}=(\mathbf{2 . 4 5 8})^{2}=\mathbf{6 . 0 4}$ |
| Upper quartile = Q3 = 7 |  |

B. Consider the frequency table

| $\mathbf{x}$ | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: |
| frequency | 3 | 5 | 7 | 5 |

to find the following:

| Mean $=\mathbf{2 7}$ | Range $=\max -\mathrm{min}=\mathbf{4 0 - 1 0 = 3 0}$ |
| :--- | :--- |
| Median $=\mathbf{3 0}$ | Interquartile range $=$ Q3-Q1 $=\mathbf{3 5}-\mathbf{2 0 = 1 5}$ |
| Mode $=\mathbf{3 0}$ | Standard deviation $=\sigma=\mathbf{1 0 . 0 5}$ |
| Lower quartile $=$ Q1 $=\mathbf{2 0}$ | Variance $=\sigma^{2}=(\mathbf{1 0 . 0 5})^{\mathbf{2}}=\mathbf{1 0 1}$ |
| Upper quartile $=$ Q3 $=\mathbf{3 5}$ |  |

C. Consider the frequency table with intervals

| $\mathbf{x}$ | $5-15$ | $15-25$ | $25-35$ | $35-45$ |
| :---: | :---: | :---: | :---: | :---: |
| frequency | 3 | 5 | 7 | 5 |

Use the midpoints (i.e. exactly the same data as above) to find:

| Mean $=\mathbf{2 7}$ | Standard deviation $=\sigma=\mathbf{1 0 . 0 5}$ |
| :--- | :--- |
| Modal group $=\mathbf{2 5 - 3 5}$ | Variance $=\sigma^{2}=(\mathbf{1 0 . 0 5})^{2}=\mathbf{1 0 1}$ |

## Notice:

For Q1,Median, Q3 we do not use the GDC. We need the cumulative frequency graph
9. REGRESSION: Use [STAT] - [CALC] - [REG] - [F1] - [F1]

## Consider the table of two variables

| $\mathbf{x}$ | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y}$ | 112 | 185 | 250 | 432 |

Find the following

| Correlation coefficient: | $\mathrm{r}=\mathbf{0 . 9 6 6}$ (i.e. strong positive correlation between x and y ) |
| :--- | :--- |
| Regression line: | $\mathrm{y}=\mathbf{1 0 . 2 5} \mathbf{x} \mathbf{- 1 1 . 5}$ |
| Prediction for $\mathrm{x}=35:$ | $\mathrm{y}=10.25 \times 35-11.5=347.25$ |

## 10. BINOMIAL DISTRIBUTION: Use [STAT] - [DIST] - [BINM]

The probability to win a game is 0.3
We play the game 12 times. So it is $B(n, p)$ with $n=12$ and $p=0.3$
Find the following probabilities

| To win | Math expression | GDC | Result |
| :--- | :---: | :--- | :--- |
| exactly 5 times | $P(X=5)$ | $\mathbf{B p d}(\mathbf{5})$ | 0.158 |
| at most 5 times | $P(X \leq 5)$ | $\mathbf{B c d}(\mathbf{0} \mathbf{- 5 )}$ | 0.882 |
| at least 5 times | $P(X \geq 5)$ | $\mathbf{B c d}(\mathbf{5} \mathbf{- 1 2 )}$ | 0.276 |
| less than 5 times | $P(X<5)$ | $\mathbf{B c d}(\mathbf{0}-\mathbf{4})$ | 0.724 |
| more than 5 times | $P(X>5)$ | $\mathbf{B c d}(\mathbf{6 - 1 2 )}$ | 0.118 |
|  | $P(3<X \leq 8)$ | $\mathbf{B c d}(\mathbf{4 - 8 )}$ | 0.506 |

Remark; Bcd(5-12) denotes Lower: 5, Upper: 12
11. NORMAL DISTRIBUTION: Use [STAT] - [DIST] - [NORM]

PROBLEM 1: FIND PROBABILITY so use [Ncd]
If mean is $\boldsymbol{\mu}=\mathbf{1 0 0 0}$ and standard deviation is $\boldsymbol{\sigma}=\mathbf{5 0}$. Find the probability

| That X is | Math notation | GDC | Result |
| :--- | :---: | :--- | :--- |
| Between 900 and 1035 | $P(900<X<1035)$ | $\mathrm{Ncd}(900-1035)$ | $\mathbf{0 . 7 3 5}$ |
| More than 1035 | $P(X>1035)$ | $\mathrm{Ncd}(1035-999999)$ | $\mathbf{0 . 2 4 2}$ |
| Less than 900 | $P(X<900)$ | $\mathrm{Ncd}(-99999-900)$ | $\mathbf{0 . 0 2 2 8}$ |
| Exactly 975 | $P(X=975)$ | $\mathbf{0}$ ("exactly" is always 0) |  |

## PROBLEM 2: PROBABILITY IS GIVEN so use [InvN]

If mean is $\boldsymbol{\mu = 1 0 0 0}$ and standard deviation is $\boldsymbol{\sigma}=\mathbf{5 0}$

| It is given | Find | Math notation | GDC | Result |
| :---: | :---: | :---: | :---: | :---: |
| Prob less than $\boldsymbol{a}$ <br> is 0.35 | $\boldsymbol{a}$ | $P(X<\boldsymbol{a})=0.35$ | InvN <br> Tail: Left, Area: $\mathbf{0 . 3 5}$ | $\boldsymbol{a}=\mathbf{9 8 1}$ |
| $42 \%$ is <br> more than $\boldsymbol{b}$ | $\boldsymbol{b}$ | $P(X>\boldsymbol{b})=0.42$ | InvN <br> Tail: Right, Area: $\mathbf{0 . 4 2}$ | $\boldsymbol{b}=\mathbf{1 0 1 0}$ |
|  | Q1 <br> Q3 | $P(\mathrm{Q} 1<X<\mathrm{Q} 3)=0.5$ | InvN <br> Tail: Central, Area: $\mathbf{0 . 5}$ | Q1=966 <br> Q3=1034 |

PROBLEM 3: $\boldsymbol{\mu}$ or $\boldsymbol{\sigma}$ or both are unknown so use the formula $\mathbf{Z}=(\mathbf{X}-\boldsymbol{\mu}) / \boldsymbol{\sigma}$ and [InvN]
Suppose that $\boldsymbol{\mu}=\mathbf{8 0 0}$ and $\boldsymbol{\sigma}$ is unknown
The information given can be expressed in different ways:

- The probability that X is less than $\mathbf{7 8 5}$ is $\mathbf{0 . 3 7}$
- $\mathbf{3 7 \%}$ is less than $\mathbf{7 8 5}$
- $\quad P(X<785)=0.37$

Answer: We use the formula

$$
Z=\frac{X-\mu}{\sigma}
$$

Right hand side: $\quad$ we know $X=785, \mu=800$
Left hand side: $\quad Z$ is obtained by the GDC:
InvN
Tail: Left
Area=0.37

$$
\begin{aligned}
& \sigma=1 \\
& \mu=0
\end{aligned}
$$

$Z=-0.332$ and hence $\sigma=45.18$

## D. EXAMPLES FOR GRAPH

12. Let $f(x)=-x^{2}+3 x+6$
(a) Solve $f(x)=0$
(i) by using [EQUA]
(ii) by using [SolveN]
(iii) by using [Graph]
(b) Solve $f(x)>0$
by using [Graph] (only)
(c) Find the range of $f$ by using [Graph]
(d) Find the area of the region R enclosed by the curve $y=f(x)$ and the x -axis
(i) by using [MATH]
(ii) by using [GRAPH]
(e) Find the volume generated when the region R is rotated $2 \pi$ rad in $x$-axis by using [MATH] (only)

## Answer

(a) $-1.372,4.372$
(b) $-1.372<x<4.372$
(c) $y \leq 8.25$
(d) 31.595
(e) 655.1
13. Let $g(x)=2 x^{3}-15 x^{2}+7 x+25$
(a) Solve $g(x)=0$
(i) by using [EQUA]
(ii) by using [SolveN]
(iii) by using [Graph]
(b) Solve $g(x)>0$
by using [Graph]
(c) Find the range of $g$
by using [Graph]
(d) Find the range of $g$ if the domain is restricted to $-2 \leq x \leq 8$
by using [Graph]
(e) Find the area of the region R enclosed by the curve $y=f(x)$ and the $x$-axis
(i) by using [MATH]
(ii) by using [GRAPH]

## Answer

(a) $-1.023,1.824,6.699$
(b) $-1.023<x<1.824, x>6.699$
(c) $y \in R$
(d) $-65.84 \leq y \leq 145$
(e) $48.448+204.081=252.529$ (or directly the result)
14. Suppose that the velocity of moving body in terms of time is given by

$$
v=2 t^{3}-15 t^{2}+7 t+25
$$

Use the graph of $g$ above (exercise 13) to find
(a) The velocity after 5 minutes
(b) The time at which the velocity is minimum
(c) The times when the direction changes
(d) the displacement from the initial position in the first 5 seconds
(e) the distance traveled in the first 5 seconds

## Answer

(a) -65
(b) $t=4.75$
(c) $t=1.824, t=6.699 \quad$ (positive roots)
(d) -100
(e) $32.437+132.436=164.873$ (or directly the result)
15. Let $f(x)=-x^{2}+3 x+6$ and $g(x)=2 x^{3}-15 x^{2}+7 x+25$
(a) Solve $f(x)=g(x)$
(i) by using [SolveN]
(ii) by using [Graph]
(b) Solve $f(x)>g(x)$ only by using graphs:
(i) by using two graphs
(ii) by using one graph only [i.e. $f(x)-g(x)$ ]
(c) Find the area enclosed by the two curves

## Answer

(a) $-0.9733,1.51,6.463$
(b) $x<-0.9733,1.51<x<6.463$
(c) 232.51
16. Let $f(x)=x^{2} e^{0.002 x} \sin 0.3 x$

Find the tangent line and the normal line at $x=2$

## Answer

You can use [Graph] - [F4:Sketch] - [Tang] or [Norm]
Remember to [SETUP] - Derivative on
We find all the details below as well as the final answers
Point $(2,2.268), m_{T}=3.2665, \quad m_{N}=-0.306$
Tangent line: $y-2.268=3.2665(x-2)$ or $y=3.2665 x-4.2654$
Normal line: $y-2.268=-0.306(x-2)$ or $y=-0.306 x+2.8798$

## E. EXAMPLES FOR RECUR

## 17. SEQUENCES: Use [RECUR]

## Consider the sequence

$$
10,13,16,19, \ldots
$$

(a) Find the $50^{\text {th }}$ term
(b) Find the sum of the first 50 terms
(c) Find the first term that exceeds 200
(d) Find the number of terms which are less than 200
(e) Find the sum of the terms which are less than 200

## Answer

The general term of this arithmetic sequence is $u_{n}=10+(n-1) \times 3$
Use the following settings
[F3:TYPE] Select F1 $a_{n}=10+(n-1) \times 3 \quad$ (use the button F1 for n )
[F5:SET] Start: 1, End : 100
In order to get the sums as well, use SETUP (shift-menu)
$\Sigma$ DISPLAY: On
Press [EXE] to get the lists for $u_{n}$ and $S_{n}$. You can find all the answers
(a) 157
(b) 4175
(c) $u_{65}=202$
(d) 64
(e) $S_{64}=6688$

## 18. PERCENTAGE GROWTH: Use [RECUR]

The populations in cities $A$ and $B$ today are 100,000 and 150,000 respectively.
They increase by $3,1 \%$ and $2,3 \%$ per year respectively
Find
(a) The population of the two cities after 12 years
(b) After how many full years the population of city A exceeds 200,000
(c) After how many full years the population of city $A$ exceeds the one of $B$.

## Answer

We use the formulas

$$
\begin{aligned}
& a_{n}=100000 \times(1.031)^{n} \\
& b_{n}=150000 \times(1.023)^{n}
\end{aligned}
$$

Remember to remove the sums and start form 0 now
SETUP (shift-menu) $\Sigma$ DISPLAY: On
[F5:SET] Start: 0, End : 100
(a) 144,246 and 197,060
(b) 23 years
(c) 53 years

