GDC REVISION for MATH SL

for the CASIO model

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From [MENU] we use

- A. RUN-MAT
- B. EQUA
- C. STAT
- D. GRAPH
- E. RECUR (for sequences)

• Remember

- o to use **RAD** or **DEG** appropriately in SETUP (shift-menu)
- o in STAT CALC to SET the first two lines

LIST1-LIST2 if you have frequencies

o in STAT – DIST we select

Data: Variable instead of List

For **NORMAL** we use $\frac{Npd}{Ncd - InvN}$ (never Npd) For **BINOM** we use $\frac{Bpd - Bcd}{Ncd - InvB}$ (never InvB)

o in SETUP (shift-menu) select Derivative On

Common mistakes

For x-3 we must use the difference – and not the symbol (-)

For $\sin^2 x$ we must write $(\sin x)^2$

For sin3(x-2) we must write sin(3(x-2))

In calculus, when we deal with sin, cos etc we must use rad

A. EXAMPLES FOR RUN-MAT

1. Use [MATH] to find the following

- (a) $log_2 5$
- (b) f'(2) and f''(2) (by using $\frac{d}{dx}$ and $\frac{d^2}{dx^2}$) for

$$f(x) = \frac{2x+5}{3x-7}$$

(c) the definite integral

$$\int_3^5 \frac{2x+5}{3x-7} dx$$

(d) the sum

$$\sum_{k=7}^{20} \frac{2k+5}{3k-7}$$

(e) the sum of the multiples of 7 between 1 and 200

Answers

- (a) 2.32
- (b) -29 and -174
- (c) 5.80
- (d) 14.07

(e) Since 200/7=28.6 and the multiples of 7 have the form 7x:

$$\sum_{x=1}^{28} 7x = 2842$$

- 2. Use [OPTION] and [PROB] to find
- (a) 12!
- (b) $\binom{12}{5}$

Answers

- (a) 479001600
- (b) 792

3. Use [OPTION] and [NUM]-[Abs] to find

(a) the definite integral

$$\int_{3}^{5} |t^2 - 16| \, dt$$

(b) the sum

$$\sum_{k=1}^{30} |100 - 6k|$$

Answers

(a) 8

4. Use [OPTION] – [CALC] – [SolveN]

to solve the equations

(a)
$$e^x = 2x + 3$$

(b)
$$sin3x = 0.5$$
 $0 \le x \le \pi$

(c)
$$sin3x = 0.5$$
 $0^{\circ} \le x \le 180^{\circ}$

Answers

(a) SolveN(
$$e^x = 2x + 3$$
) gives: -1,37, 1,92

Notice: We can <u>add a restriction</u> for the domain. Use rad or degrees appropriately:

(b) SolveN(
$$sin3x = 0.5$$
, x, 0, π) gives: $\pi/18$, $5\pi/18$, $13\pi/18$, $17\pi/18$

(c) SolveN(
$$sin3x = 0.5$$
, x, 0,180) gives: 10, 50, 130, 170

B. EXAMPLES FOR EQUA

5. Use **[F2:Polynomial]** to solve
$$x^3 - 5x^2 + 3x + 6 = 0$$

Answer

6. Use [F1:Simultaneous] to solve the simultaneous equations

$$3x + 5y = 18$$

$$7x - 4y = -5$$

Answer

$$x = 1, y = 3$$

7. Use [F1:Simultaneous] to solve the simultaneous equations

$$3a + 6b - c = 19$$

$$a - 2b + 4c = 3$$

$$7a - 13c = -19$$

Answer

$$a = 1, b = 3, c = 2$$

C. EXAMPLES FOR STAT

8. STATISTICS: Use [STAT] - [CALC] - [SET] - [VAR1]

A. Consider the data

2, 5, 7, 5, 3, 2, 5, 1, 7, 9

to find the following:

Mean = 4.6	Range = max-min = 9-1 =8
Median = 5	Interquartile range = Q3 – Q1 = 7-2=5
Mode = 5	Standard deviation = σ = 2.458
Lower quartile = Q1 = 2	Variance = σ^2 = (2.458) ² = 6.04
Upper quartile = Q3 = 7	

B. Consider the frequency table

х	10	20	30	40
frequency	3	5	7	5

to find the following:

Mean = 27	Range = max-min = 40-10=30
Median = 30	Interquartile range = Q3 – Q1 = 35-20=15
Mode = 30	Standard deviation = σ = 10.05
Lower quartile = Q1 = 20	Variance = σ^2 = (10.05) ² = 101
Upper quartile = Q3 = 35	

C. Consider the frequency table with intervals

x	5-15	15-25	25-35	35-45
frequency	3	5	7	5

Use the midpoints (i.e. exactly the same data as above) to find:

Mean = 27	Standard deviation = σ = 10.05
Modal group = 25-35	Variance = σ^2 = (10.05) ² = 101

Notice:

For Q1, Median, Q3 we do not use the GDC. We need the cumulative frequency graph

9. REGRESSION: Use [STAT] - [CALC] - [REG] - [F1] - [F1]

Consider the table of two variables

х	10	20	30	40
У	112	185	250	432

Find the following

Correlation coefficient: r = 0.966 (i.e. strong positive correlation between x and y)

Regression line: y=10.25x-11.5

Prediction for x=35: $y = 10.25 \times 35 - 11.5 = 347.25$

10. BINOMIAL DISTRIBUTION: Use [STAT] - [DIST] - [BINM]

The probability to win a game is 0.3

We play the game 12 times. So it is B(n,p) with n=12 and p=0.3

Find the following probabilities

To win	Math expression	GDC	Result
exactly 5 times	P(X=5)	Bpd(5)	0.158
at most 5 times	$P(X \le 5)$	Bcd(0-5)	0.882
at least 5 times	$P(X \ge 5)$	Bcd(5-12)	0.276
less than 5 times	P(X < 5)	Bcd(0-4)	0.724
more than 5 times	P(X > 5)	Bcd(6-12)	0.118
	$P(3 < X \le 8)$	Bcd(4-8)	0.506

Remark; Bcd(5-12) denotes Lower: 5, Upper: 12

11. NORMAL DISTRIBUTION: Use [STAT] - [DIST] - [NORM]

PROBLEM 1: FIND PROBABILITY so use [Ncd]

If mean is μ =1000 and standard deviation is σ =50. Find the probability

That X is	Math notation	GDC	Result
Between 900 and 1035	P(900 < X < 1035)	Ncd(900-1035)	0.735
More than 1035	P(X > 1035)	Ncd(1035-999999)	0.242
Less than 900	P(X < 900)	Ncd(-99999–900)	0.0228
Exactly 975	P(X=975)	0 ("exactly" is always 0)	

PROBLEM 2: PROBABILITY IS GIVEN so use [InvN]

If mean is μ =1000 and standard deviation is σ =50

It is given	Find	Math notation	GDC	Result
Prob less than <i>a</i>	а	P(X < a) = 0.35	InvN	a =981
is 0.35		1 (11 1 11)	Tail: Left, Area: 0.35	u 001
42% is	ь	$P(X > \boldsymbol{b}) = 0.42$	InvN	<i>b</i> =1010
more than b		1 (11 / 1) 1111	Tail: Right, Area: 0.42	2 2020
	Q1	P(Q1 < X < Q3) = 0.5	InvN	Q1= 966
	Q3	(4= (40) 0.0	Tail: Central, Area: 0.5	Q3= 1034

<u>PROBLEM 3:</u> μ or σ or both are unknown so use the formula **Z=(X-\mu)/\sigma** and [InvN]

Suppose that $\mu=800$ and σ is unknown

The information given can be expressed in different ways:

- The probability that X is less than **785** is **0.37**
- 37% is less than 785
- P(X < 785) = 0.37

Answer: We use the formula

$$Z = \frac{X - \mu}{\sigma}$$

Right hand side: we know X = 785, $\mu = 800$ Left hand side: Z is obtained by the GDC:

InvN

Tail: Left

Area=0.37

σ=1

μ=0

Z=-0.332 and hence $\sigma=45.18$

D. EXAMPLES FOR GRAPH

12. Let $f(x) = -x^2 + 3x + 6$

- (a) Solve f(x) = 0
 - (i) by using [EQUA]
 - (ii) by using [SolveN]
 - (iii) by using [Graph]
- (b) Solve f(x) > 0

by using [Graph] (only)

(c) Find the range of *f*by using [Graph]

- (d) Find the area of the region R enclosed by the curve y = f(x) and the x-axis
 - (i) by using [MATH]
 - (ii) by using [GRAPH]
- (e) Find the volume generated when the region R is rotated 2π rad in x-axis by using [MATH] (only)

Answer

(a) -1.372, 4.372 (b) -1.372 < x < 4.372 (c) $y \le 8.25$ (d) 31.595 (e) 655.1

13. Let $g(x) = 2x^3 - 15x^2 + 7x + 25$

- (a) Solve g(x) = 0
 - (i) by using [EQUA]
 - (ii) by using [SolveN]
 - (iii) by using [Graph]
- (b) Solve g(x) > 0

by using [Graph]

(c) Find the range of g

by using [Graph]

- (d) Find the range of g if the domain is restricted to $-2 \le x \le 8$ by using [Graph]
- (e) Find the area of the region R enclosed by the curve y = f(x) and the x-axis
 - (i) by using [MATH]
 - (ii) by using [GRAPH]

Answer

- (a) -1.023, 1.824, 6.699 (b) -1.023 < x < 1.824, x > 6.699 (c) $y \in R$
- (d) $-65.84 \le y \le 145$ (e) 48.448 + 204.081 = 252.529 (or directly the result)

14. Suppose that the velocity of moving body in terms of time is given by

$$v = 2t^3 - 15t^2 + 7t + 25$$

Use the graph of g above (exercise 13) to find

- (a) The velocity after 5 minutes
- (b) The time at which the velocity is minimum
- (c) The times when the direction changes
- (d) the displacement from the initial position in the first 5 seconds
- (e) the distance traveled in the first 5 seconds

Answer

- (a) -65 (b) t = 4.75 (c) t = 1.824, t = 6.699 (positive roots)
- (d) -100 (e) 32.437 + 132.436 = 164.873 (or directly the result)

15. Let
$$f(x) = -x^2 + 3x + 6$$
 and $g(x) = 2x^3 - 15x^2 + 7x + 25$

- (a) Solve f(x) = g(x)
 - (i) by using [SolveN]
 - (ii) by using [Graph]
- (b) Solve f(x) > g(x) only by using graphs:
 - (i) by using two graphs
 - (ii) by using one graph only [i.e. f(x) g(x)]
- (c) Find the area enclosed by the two curves

Answer

- (a) -0.9733, 1.51, 6.463 (b) x < -0.9733, 1.51 < x < 6.463
- (c) 232.51

16. Let
$$f(x) = x^2 e^{0.002x} sin 0.3x$$

Find the tangent line and the normal line at x = 2

Answer

You can use [Graph] - [F4:Sketch] - [Tang] or [Norm]

Remember to [SETUP] - Derivative on

We find all the details below as well as the final answers

Point (2, 2.268),
$$m_T = 3.2665$$
, $m_N = -0.306$

Tangent line:
$$y - 2.268 = 3.2665 (x - 2)$$
 or $y = 3.2665 x - 4.2654$

Normal line:
$$y - 2.268 = -0.306 (x - 2)$$
 or $y = -0.306x + 2.8798$

E. EXAMPLES FOR RECUR

17. SEQUENCES: Use [RECUR]

Consider the sequence

- (a) Find the 50th term
- (b) Find the sum of the first 50 terms
- (c) Find the first term that exceeds 200
- (d) Find the number of terms which are less than 200
- (e) Find the sum of the terms which are less than 200

Answer

The general term of this arithmetic sequence is $u_n = 10 + (n-1) \times 3$

Use the following settings

[F3:TYPE] Select **F1**
$$a_n = 10 + (n-1) \times 3$$
 (use the button F1 for n)

[F5:SET] Start: 1, End: 100

In order to get the sums as well, use SETUP (shift-menu)

Σ DISPLAY: On

Press [EXE] to get the lists for u_n and S_n . You can find all the answers

(a) 157 (b) 4175 (c)
$$u_{65} = 202$$
 (d) 64 (e) S_{64} =6688

18. PERCENTAGE GROWTH: Use [RECUR]

The populations in cities A and B today are 100,000 and 150,000 respectively.

They increase by 3,1% and 2,3% per year respectively

Find

- (a) The population of the two cities after 12 years
- (b) After how many full years the population of city A exceeds 200,000
- (c) After how many full years the population of city A exceeds the one of B.

Answer

We use the formulas

$$a_n = 100000 \times (1.031)^n$$

$$b_n = 150000 \times (1.023)^n$$

Remember to remove the sums and start form 0 now

SETUP (shift-menu) Σ DISPLAY: On

[F5:SET] Start: 0, End: 100

(a) 144,246 and 197,060 (b) 23 years (c) 53 years