**HAEF IB – FURTHER MATH HL**

|  |  |
| --- | --- |
| **P1:\_\_\_/35** | **P2:\_\_\_/35** |

**Total: \_\_\_\_\_%**

**Grade:**

**TEST 3**

**Sets, Groups and Relations**

**Paper 1**

*by Christos Nikolaidis*

**Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Date: 25/1/2017**

**Questions**

1. *[Maximum mark: 5]*
2. Show by means of a Venn diagram that * [1 mark]*
3. Using (a) and set algebra, prove that * [4 marks]*
4. *[Maximum mark: 7]*

Consider the function  given by



1. Show that  is a bijection *[6 marks]*
2. Find  *[1 mark]*
3. *[Maximum mark: 8]*

Consider the functions** and **. Given that 

is a bijection, show that

1. *f* is an injection *[3 marks]*
2. *g* is a surjection *[3 marks]*
3. *f* and *g* are not necessarily bijections. *[2 marks]*
4. *[Maximum mark: 15]*

Let *D* =**and ** a function given by

**

1. Explain why  is a bijection. *[2 marks]*
2. Show that  is self-inverse *[2 marks]*
3. Let T be a relation on *D* given by

** if and only if **

Determine whether *T* is reflexive, symmetric or transitive. *[5 marks]*

1. Let *S* be a relation on *D* × *R* such that

** if and only if **

1. Show that *S* is an equivalence relation.
2. Describe the equivalence classes of *S* (i.e. the partition of *D* × *R*) *[6 marks]*

**HAEF IB – FURTHER MATH HL**

**TEST 3**

**Sets, Groups and Relations**

**Paper 2**

*by Christos Nikolaidis*

**Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Date: 25/1/2017**

**Questions**

1. *[Maximum mark: 15]*

Consider the binary operation



on the set of non-zero real numbers .

*[2 marks]*

*[5 marks]*

*[2 marks]*

*[6 marks]*

1. Show that (,) has an identity element *a* and state its value.
2. Show that (,) is an Abelian group.

Consider also a homomorphism



where (,+) is the standard additive group.

1. Show that**.
2. Given that , where  is a positive integer

(i) find the value , by using (c)

(ii) confirm that  is a homomorphism;

(iii) explain why  is not an isomorphism;

(iv) find the kernel .

(v) Describe the cosets of 

1. *[Maximum mark: 20]*

Consider the multiplicative group (), where  and 

*[4 marks]*

*[3 marks]*

*[6 marks]*

*[4 marks]*

*[2 marks]*

*[1 marks]*

is the multiplication of integers modulo 7.

1. Write down the Cayley table of this group.
2. Show that () is cyclic and find its smallest generator.

Consider also the additive group , where  and  is

the addition of integers modulo 6.

1. If *f* is a homomorphism from () to , with 

(i) Find the value of  by using the fact 

(ii) Copy and complete the following tables by applying *f* on the powers of 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 0 |  | 1 |  |  |  |

1. If *g* is a homomorphism from () to , with , copy

and complete the following table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
|  |  |  | 2 |  |  |  |

1. Determine which of the two functions *f* , *g* is an isomorphism. Explain.
2. Write down the kernel of *g*.