**HAEF IB – FURTHER MATH HL**

**TEST 2**

**Linear Algebra**

*by Christos Nikolaidis*

**Marks:\_\_\_\_/100**

**Grade: \_\_\_\_\_\_**

**Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Date:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Questions**

1. *[maximum mark: 6]*

Let .

*[4 marks]*

*[2 marks]*

1. Show that  is a subspace of .
2. Find a basis of and state its dimension.
3. *[maximum mark: 6]*

Show that

*[3 marks]*

*[3 marks]*

1. , ,  are linearly dependent vectors (by using the definition)
2. ,  form a basis for .
3. *[maximum mark: 7]*

Let be two vectors in . Show that

*[2 marks]*

*[5 marks]*

1. ,0 are linearly dependent (by using the definition)
2.  and  are linearly independent **if and only if**  and  are linearly independent
3. *[maximum mark: 6]*

*[4 marks]*

*[2 marks]*

1. If  and  are subspaces of  show that  is also a subspace of .
2. By selecting appropriate subspaces  and  of  show  is not

necessarily a subspace of .

1. *[maximum mark: 13]*

Let be a 2x2 matrix.

1. Show that the characteristic polynomial is 

*[2 marks]*

*[4 marks]*

*[2 marks]*

*[2 marks]*

*[3 marks]*

1. Show that the matrix A is a root of the corresponding matrix polynomial,

that is  (**Cayley**–**Hamilton** theorem)

Let now 

1. Use result (b) in order toexpress  as a linear combination of *A* and *I*.
2. Express  in terms of  and only.
3. Express in terms of *A* and *I* only

1. *[maximum mark: 14]*

Let 

*[3 marks]*

*[1 mark]*

*[3 marks]*

*[2 marks]*

*[3 marks]*

*[2 marks]*

1. Find the row rank of *A.*
2. State the column rank of *A.*
3. Describe the row space of *A,* in terms of a basis.
4. Describe the column space when *a* = 6.
5. Find the dimension of the null space of *A* and **hence** the null space.
6. Find the dimension of the null space of *AT*.
7. *[maximum mark: 6 ]*

Let  be a subspace of . Show that

 form a basis for  (i.e.  are linearly independent and span ).

**if and only if**

any vector in can be expressed **uniquely** as a linear combination of 

1. *[maximum mark: 9]*

Let  given by 

*[3 marks]*

*[2 marks]*

*[2 marks]*

*[2 marks]*

1. Show that  is a linear transformation
2. Find the kernel of *T*.
3. Write down the range of *T*.
4. Find the standard matrix corresponding to *T*
5. *[maximum mark: 13]*

Let  a linear transformation, where ***0n*** and ***0m***are the corresponding zero vectors

*[3 marks]*

*[5 marks]*

*[5 marks]*

1. Show T(***0n***) = ***0m***
2. Show that the kernel, ker*T,* of the transformation is a subspace of .
3. Show that *T* is one-to-one if and only if ker *T* ={***0n*** }.
4. *[maximum mark: 5]*

Let  be a linear transformation given by 

Find the image of the straight line .

1. *[maximum mark: 15]*

Let .

1. Show that and hence find formulas for

*[4 marks]*

*[8 marks]*

*[3 marks]*

1.  (even powers)
2.  (odd powers)
3. Diagonalise the matrix *A* by using its eigenvalues and **hence** show that



1. Confirm that (a) and (b) give the same result for 